Agarra la Onda!⚽

Gravitational Waves from Core Collapse Supernovae

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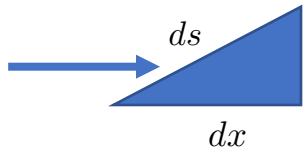
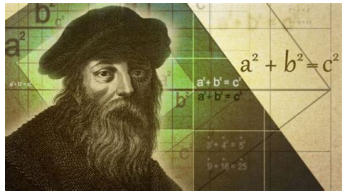
Infinitesimal segment along an arbitrary path in 2D Euclidean space.

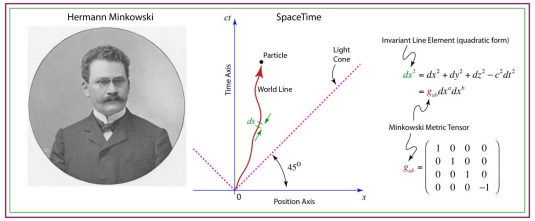


$$ds^2 = dx^2 + dy^2 = \delta_{ij} dx^i dx^j = \begin{pmatrix} dx & dy \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$



$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu = \begin{pmatrix} dt & dx & dy & dz \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

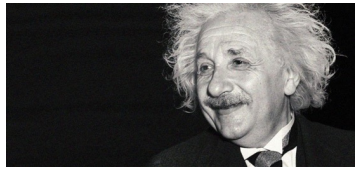
Minkowski Spacetime



Minkowski Metric

General Spacetime $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

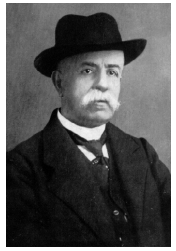
Einstein Equations $\mathbf{G} = \mathbf{8\pi T}$



Einstein Tensor $G_{\mu\nu} = 8\pi T_{\mu\nu}$

Stress-Energy Tensor

General Relativity in a Word

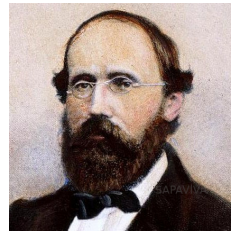


$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Ricci Tensor $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$

$$R^\lambda_{\mu\beta\nu} = \partial_\beta \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\beta} + \Gamma^\lambda_{\alpha\beta} \Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\beta} \Gamma^\alpha_{\mu\beta}$$

Riemann Curvature Tensor

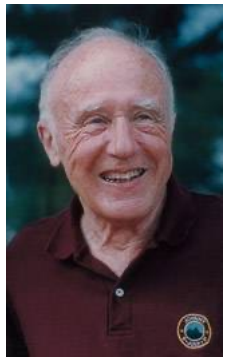


Christoffel Symbols $\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\beta} \left(\partial_\nu g_{\mu\rho} + \partial_\mu g_{\nu\rho} - \partial_\rho g_{\mu\rho} \right)$

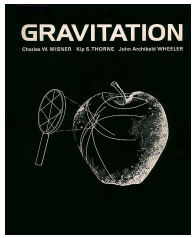
$$\mathbf{G} = 8\pi\mathbf{T}$$

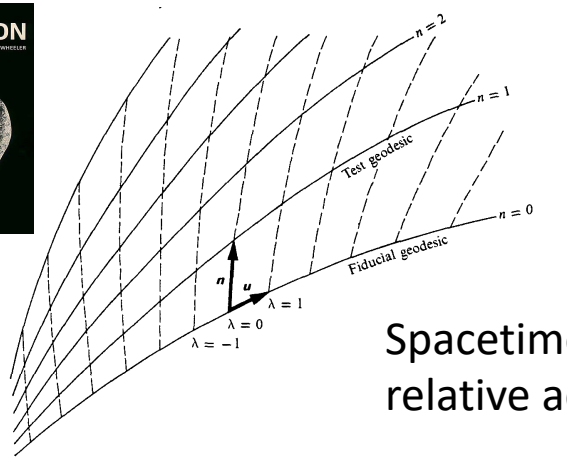
"Spacetime tells matter how to move."

"Matter tells spacetime how to curve."



John Archibald Wheeler



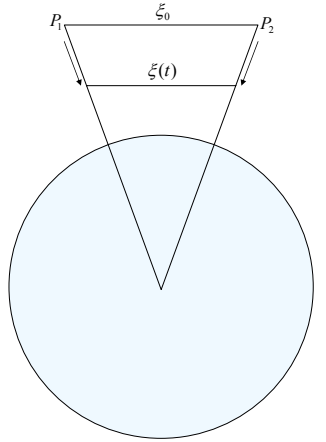


$$\frac{D^2 n^\alpha}{d\lambda^2} = -R^\alpha_{\beta\gamma\delta} u^\beta n^\gamma u^\delta$$

Spacetime curvature gives rise to the relative acceleration of geodesics.

$$\frac{\partial^2 \xi^i}{\partial t^2} = -\frac{\partial^2 \Phi}{\partial x^i \partial x^j}\xi^j$$

What we would get by purely Newtonian considerations.



Newtonian Gravity in Einstein's Language:

$$ds^2 = -(1+2\Phi)dt^2 + dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$$


Canon
McDonald's

Gravitational Waves

Given the physicality of spacetime in Einsteinian gravity, can it support waves?

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad \Rightarrow \quad \Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

$$\Box h_{\mu\nu} = 0$$

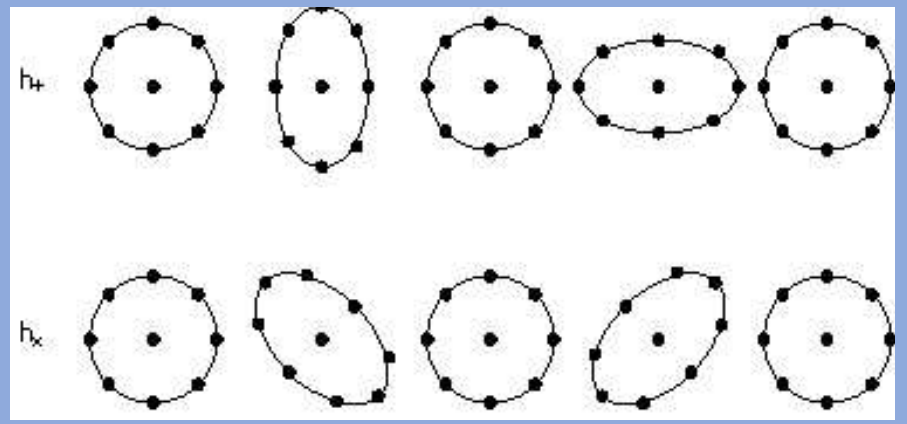
Vacuum Case

$$h_{\mu\nu} = e_{\mu\nu} e^{ik\cdot x}$$

Polarization Tensor (elements are constant in this case)

$$h_{ij}^{TT} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{ik\cdot x}$$

$$k = (k_0, 0, 0, k_3)$$



VACUUM SOLUTION

Kotake et al. 2006 *Rep. Prog. Phys.* **69** 971

Gravitational Waves, Continued

$$\Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = -16\pi \int d^4x' G(x-x') T_{\mu\nu}(x')$$

$$\Box G(x-x') = \delta^{(4)}(x-x')$$

$$G(x-x') = -\frac{1}{4\pi|\vec{x}-\vec{x}'|}\delta(t-|\vec{x}-\vec{x}'|-t')$$

$$\bar{h}_{\mu\nu}(\vec{x},t) = 4\int d^3x' \frac{T_{\mu\nu}(\vec{x}', t-|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|}$$

$$h_+ = \frac{h_{\theta\theta}}{r^2}$$

$$h_\times = \frac{h_{\theta\phi}}{r^2 sin\theta}$$

Now with sources.

Kotake et al. 2006 *Rep. Prog. Phys.* **69** 971

$$X_{lm} = 2\frac{\partial}{\partial\phi}\left(\frac{\partial}{\partial\theta} - \cot\theta\right) Y_{lm}(\theta,\phi)$$

$$W_{lm} = \left(\frac{\partial^2}{\partial\theta^2} - \cot\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right) Y_{lm}(\theta,\phi)$$

Slow Motion Approximation

Expand $T_{\mu\nu}$ in powers of $\frac{v}{c}$.

Transverse-Traceless Decomposition

$$f_{ij}^{lm} = r^2 \begin{pmatrix} r & \theta & \phi \\ 0 & 0 & 0 \\ 0 & W_{lm} & X_{lm} \\ 0 & X_{lm} & -\sin^2\theta W_{lm} \end{pmatrix}$$

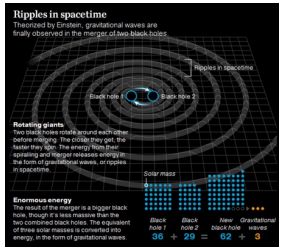
$$h_{ij}^{\mathrm{TT}} = \frac{G}{c^4}\frac{1}{r}\sum_{m=-2}^{+2}\frac{d^2 I_{2m}}{dt^2}(t-\frac{r}{c})f_{ij}^{2m} \qquad I_{2m} = \frac{16\sqrt{3}\pi}{15}\int \tau_{00} Y_{2m}^* r^2 dV$$

What causes such waves?

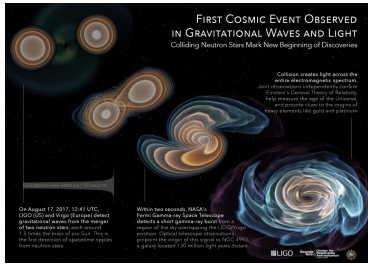
$$h_{ij}^{\mathrm{TT}} = \frac{G}{c^4}\frac{1}{r}\sum_{m=-2}^{+2}\frac{d^2 I_{2m}}{dt^2}(t-\frac{r}{c})f_{ij}^{2m}$$

astrophysical phenomena with time-dependent quadrupole moments

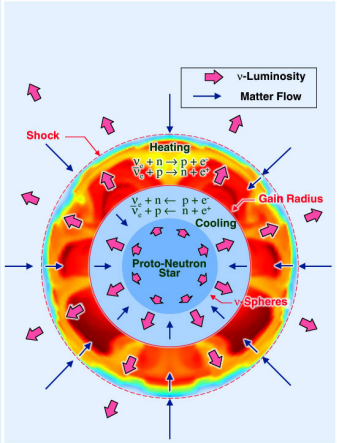
Binary Black Hole Mergers



Neutron Star Mergers



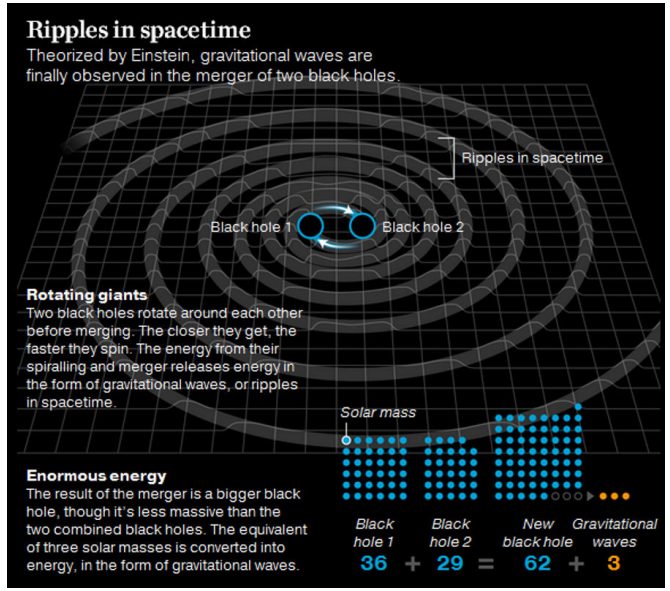
Core Collapse Supernovae



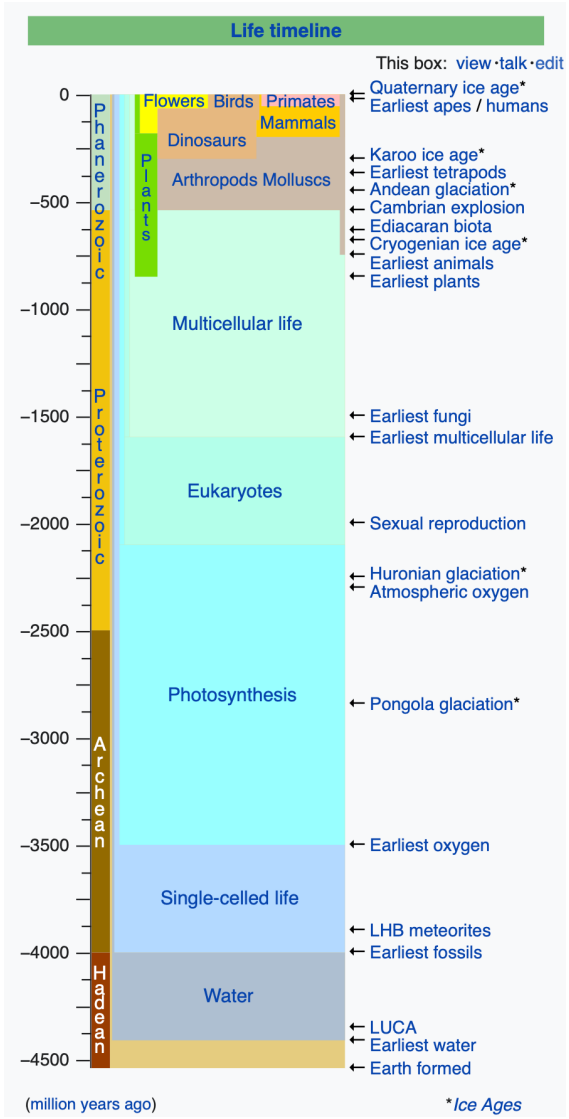
What causes such waves?

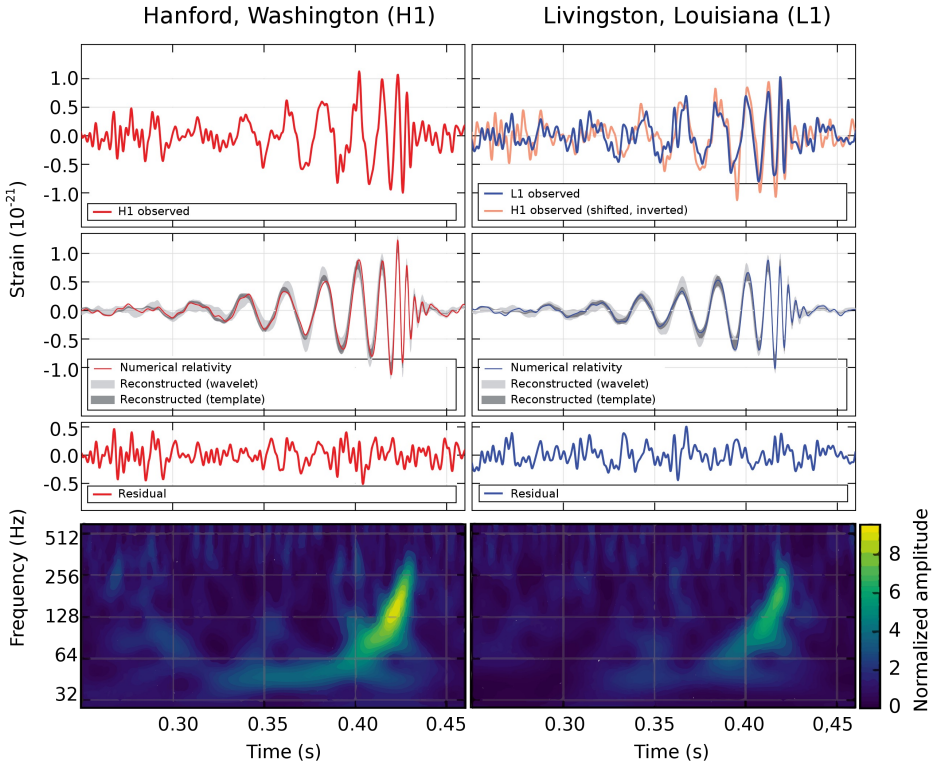
Binary Black Hole Mergers

GW150914



@1.3 billion lyr!

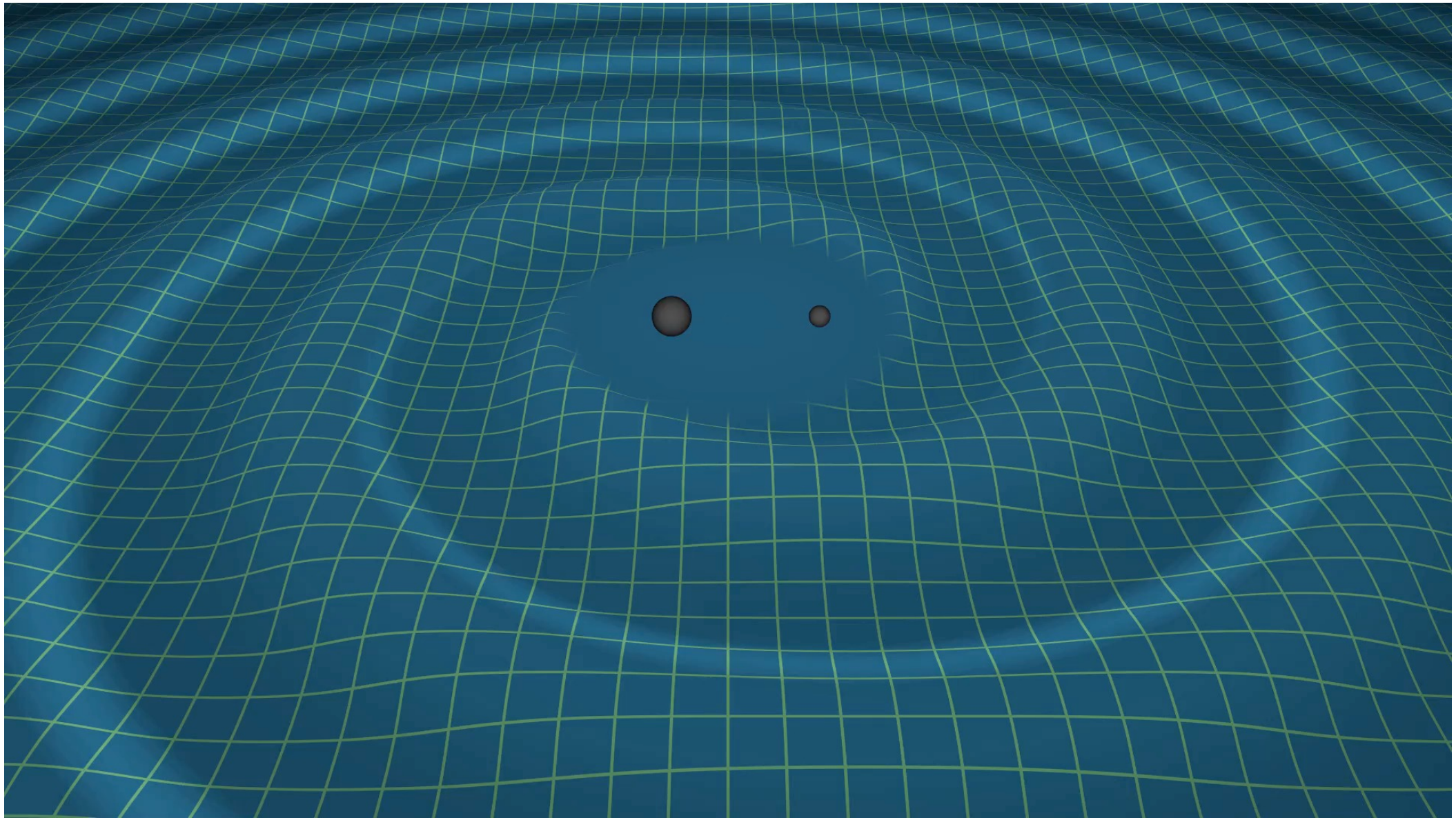




Abbott et al. 2016 PRL 116 061102

Time delay between Hanford and Livingston detections consistent with gravitational wave propagation at the speed of light.

Gravitational Waves from Compact Binary Mergers: The Movie

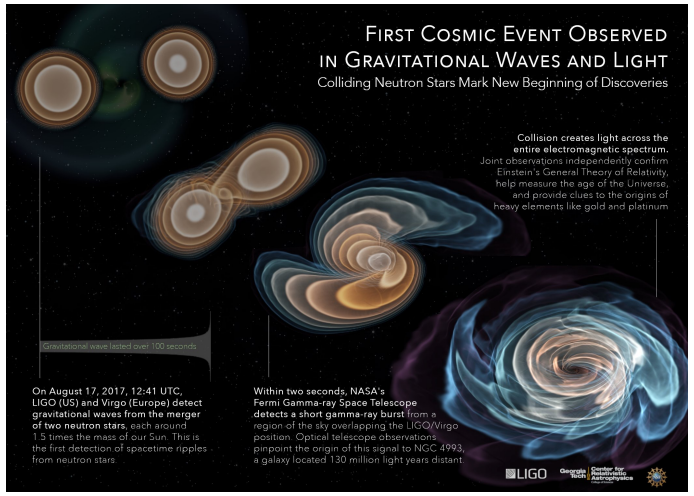


What causes such waves?

GW170817

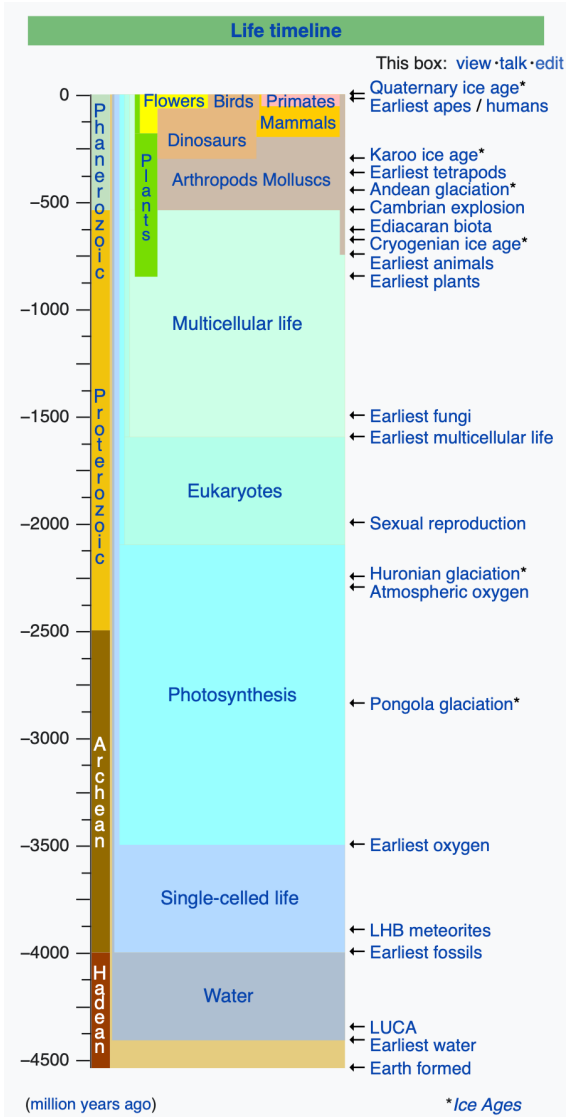
GRB170817A

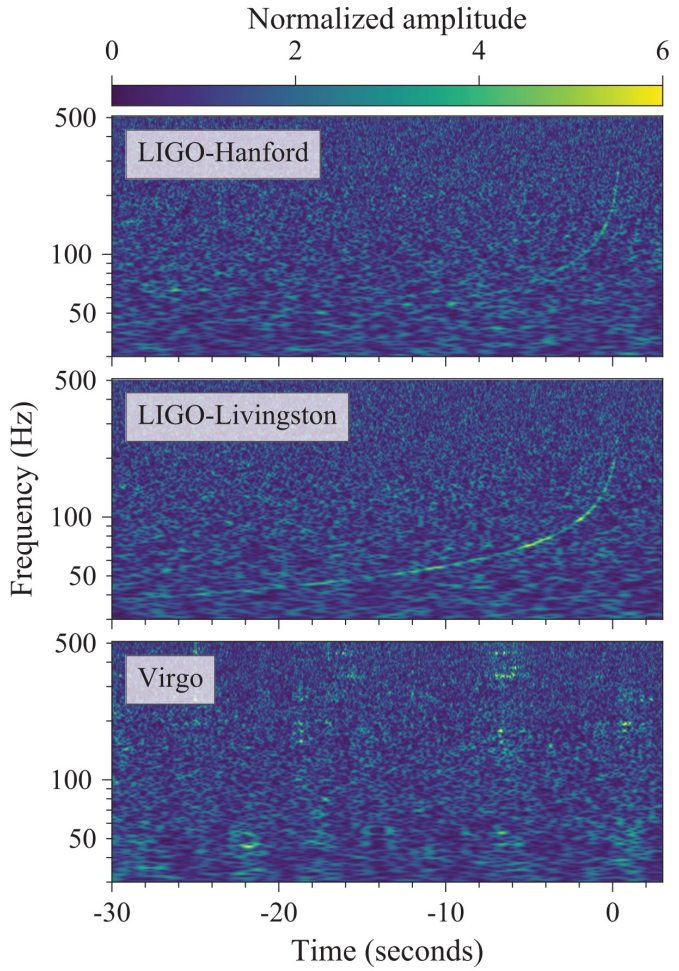
Binary Neutron Star Mergers



@150 million lyr

Observed by 70 observatories on 7 continents.





Abbott et al. 2017 PRL 119 161101

Gravitational Wave Telescopes

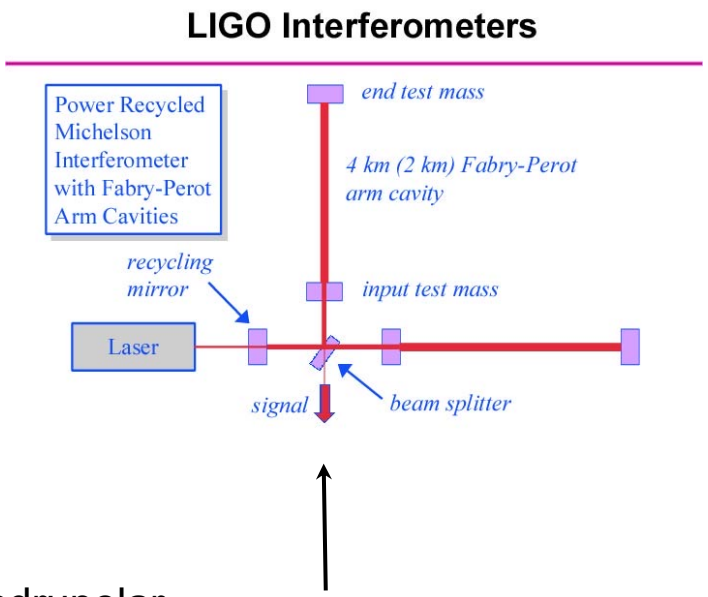
Laser Interferometer Gravitational Wave Observatory (LIGO)

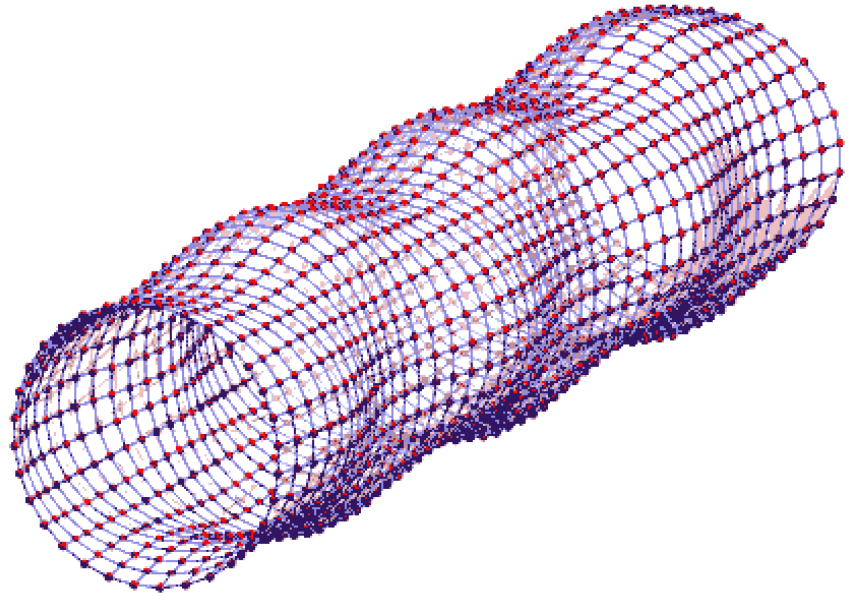
LIGO Hanford

LIGO Livingston

Gertsenshtein, M.E. (1962). "Wave Resonance of Light and Gravitational Waves". *Soviet Physics – Journal of Experimental and Theoretical Physics*. **14**: 84.

Weiss, Rainer (1972). "Electromagnetically coupled broadband gravitational wave antenna". *Quarterly Progress Report of the Research Laboratory of Electronics*. **105** (54): 84.

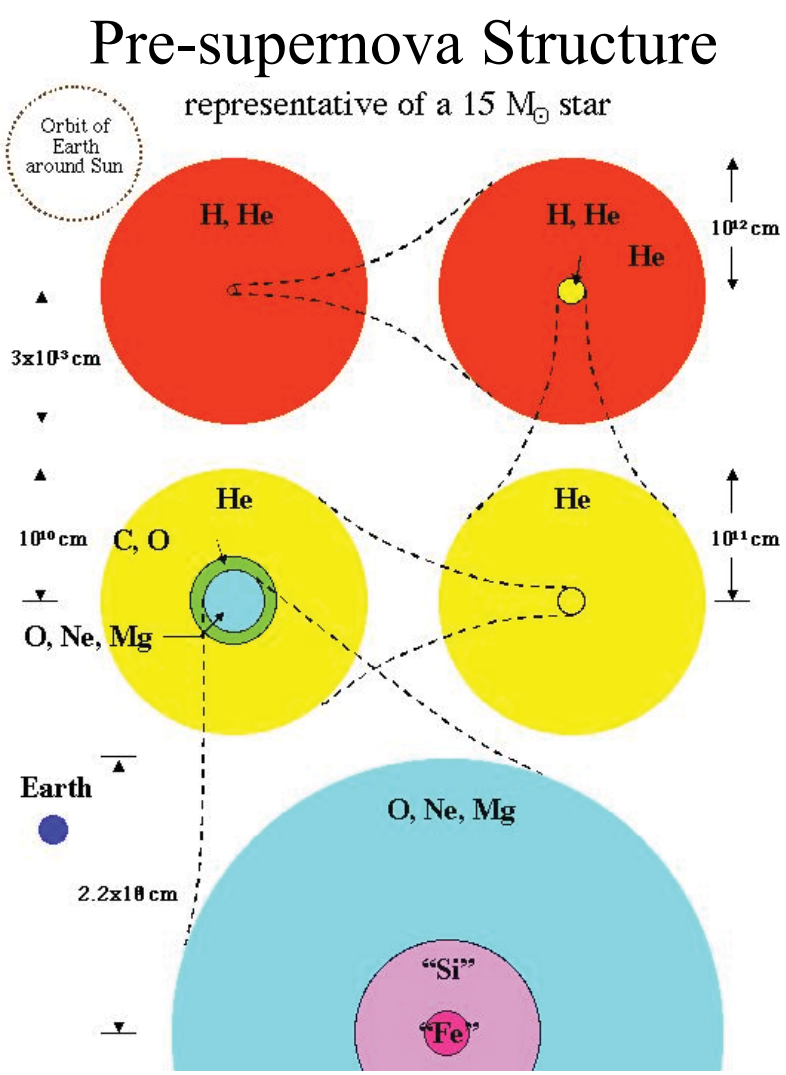


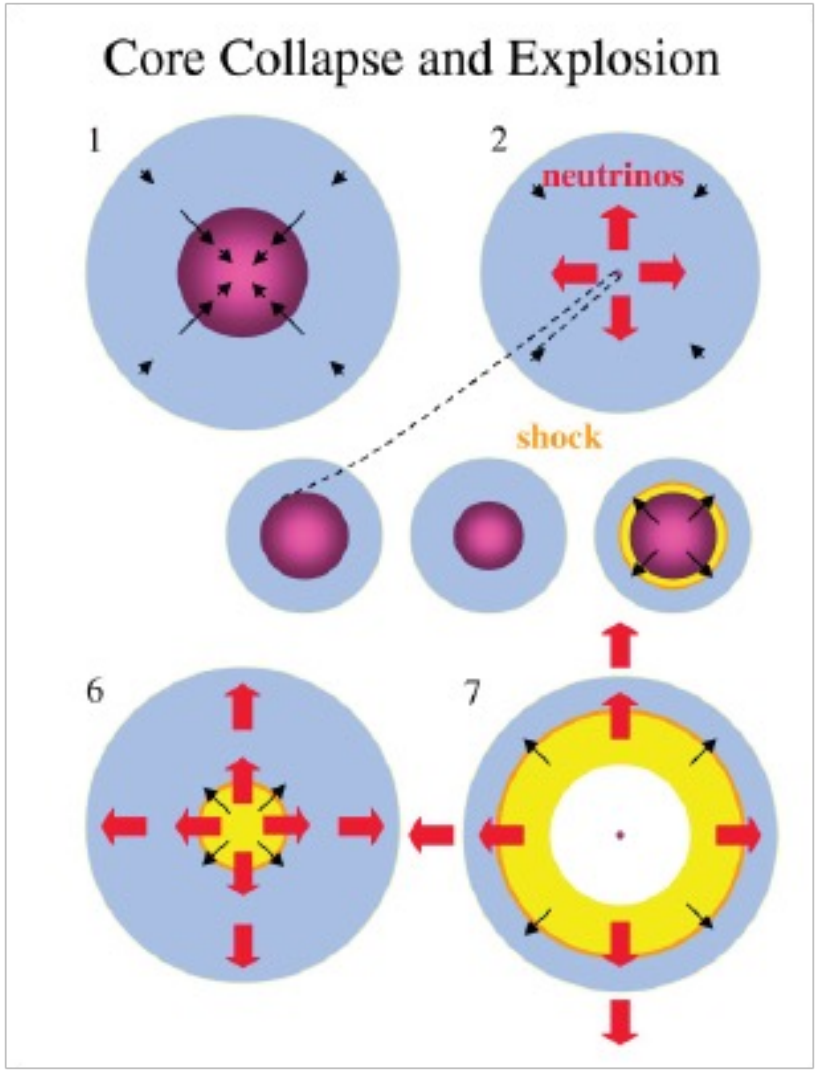


Gravitational waves are quadrupolar.

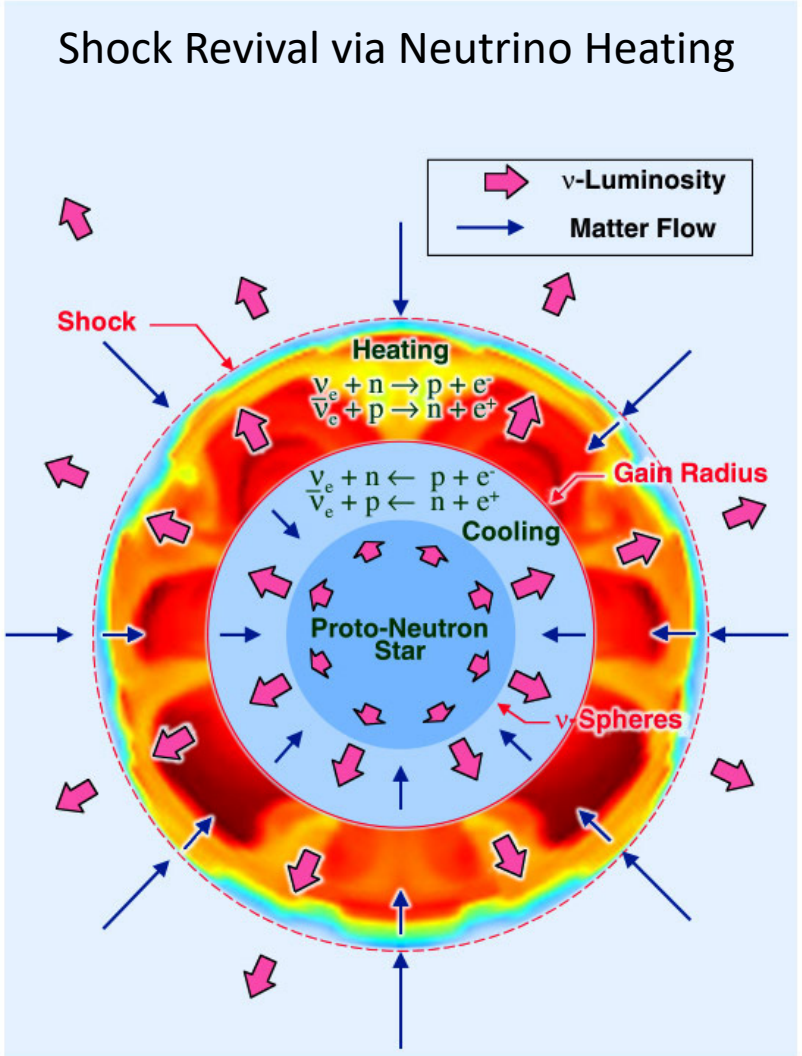
In GW150914, interferometer arm length changed by 1/1000 the diameter of a proton!

Core Collapse Supernovae: Stages to Catastrophe



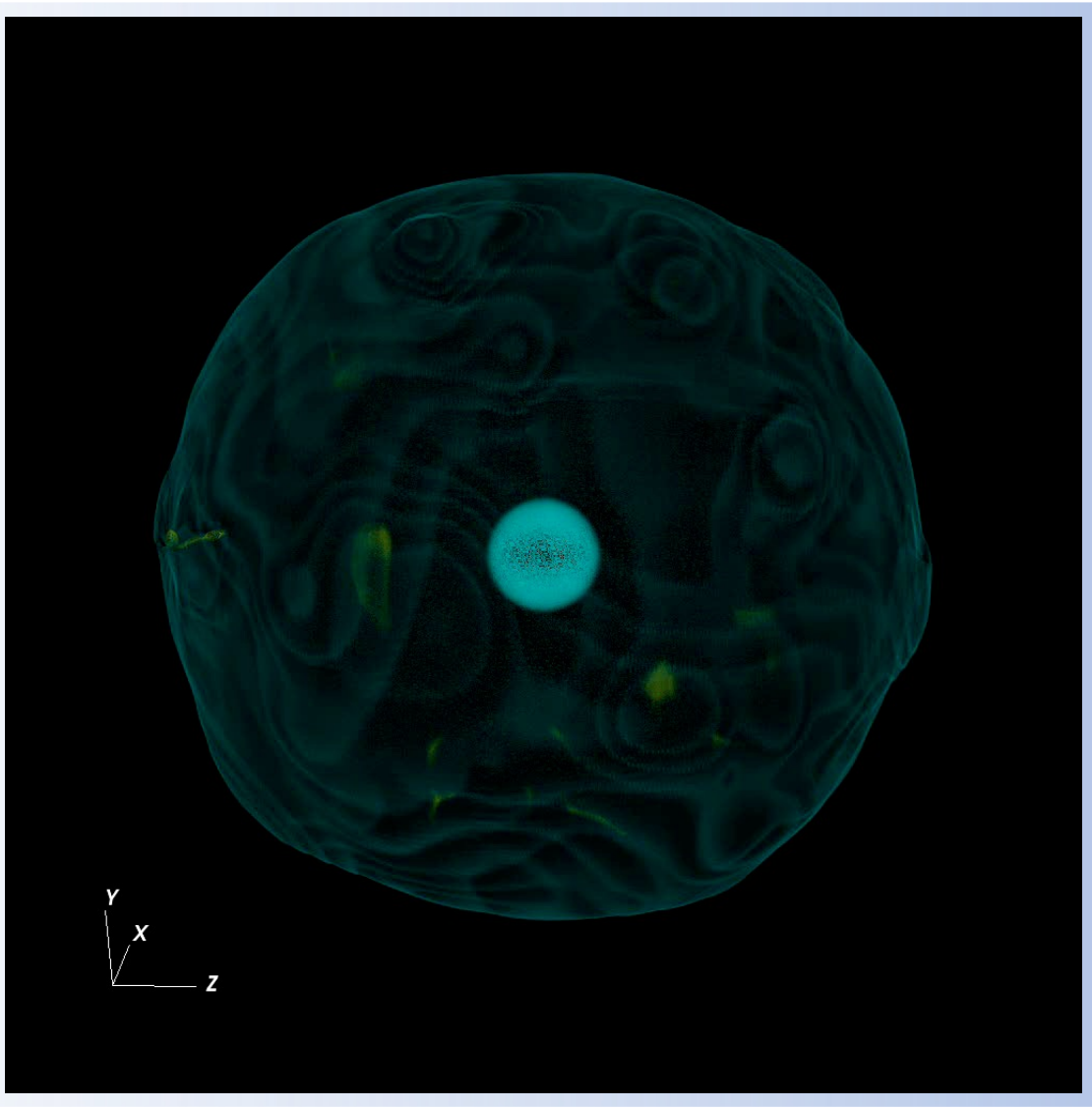


Shock Stall



+ SASI!

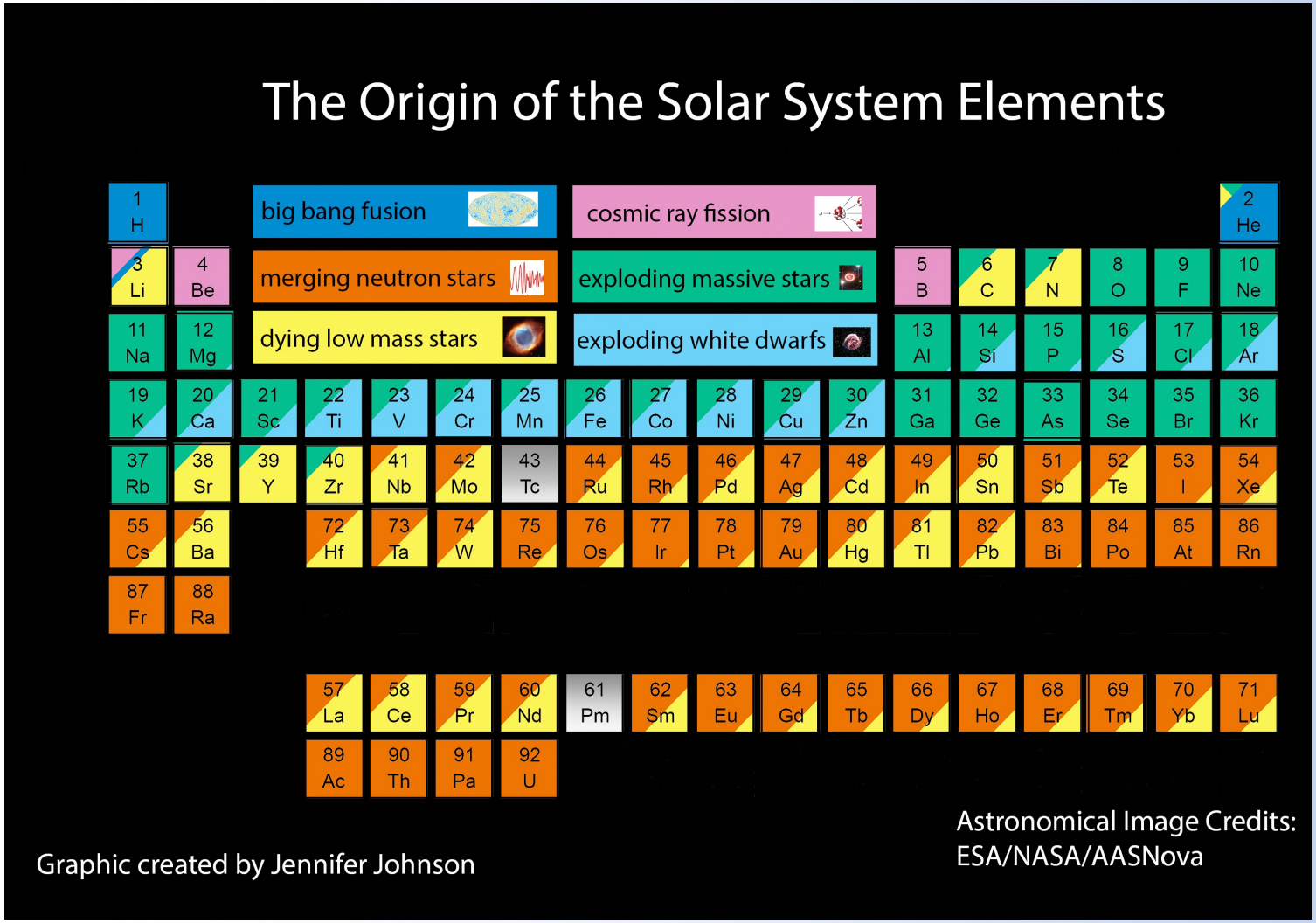
Boiling a Star with Neutrinos



One of the first published realistic 3D core collapse supernova models.

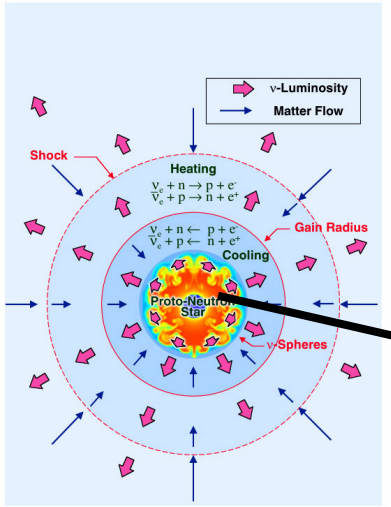
Lentz et al. 2015 *Ap.J. Lett.* **807** 31

Why We Care

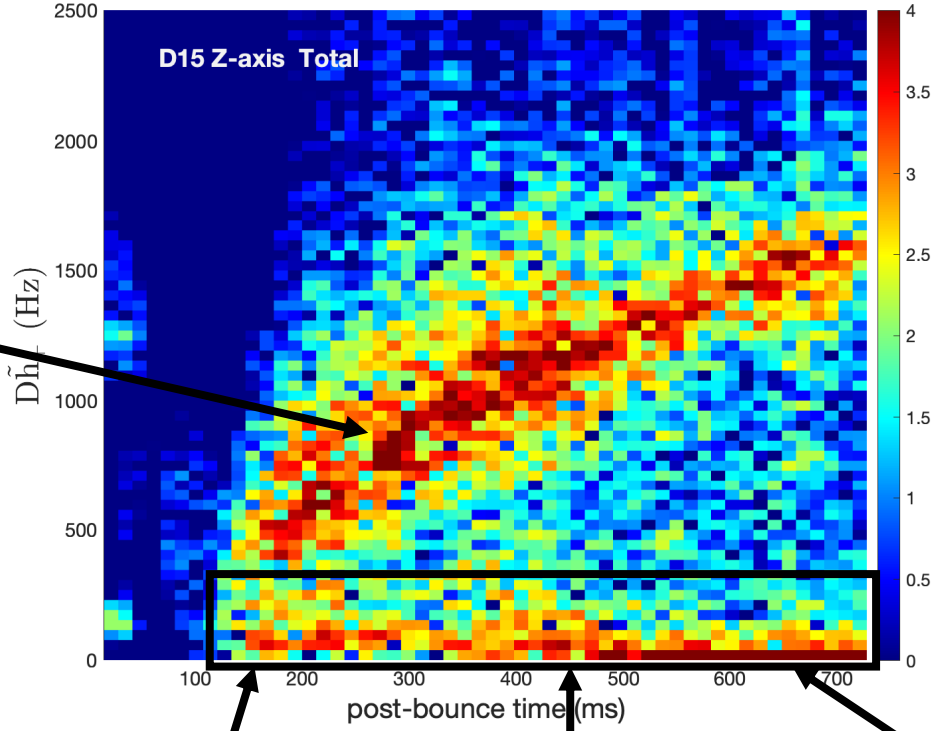


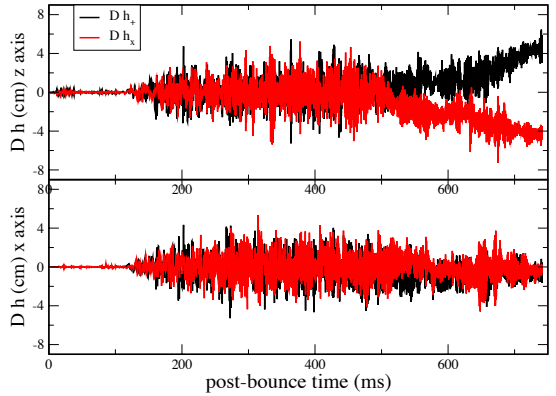
Graphic created by Jennifer Johnson

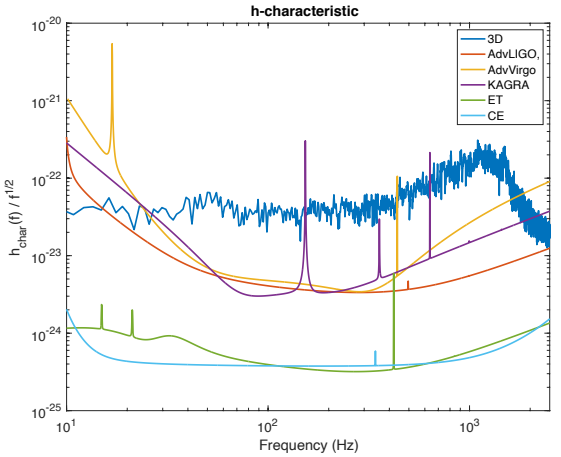
Astronomical Image Credits:
ESA/NASA/AASNova

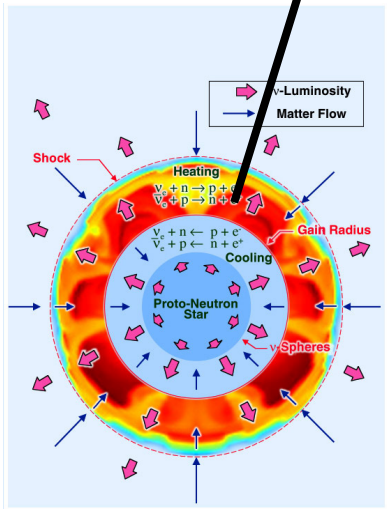


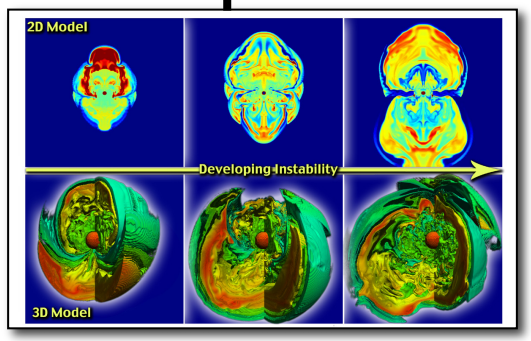
Sources of CCSN Gravitational Waves

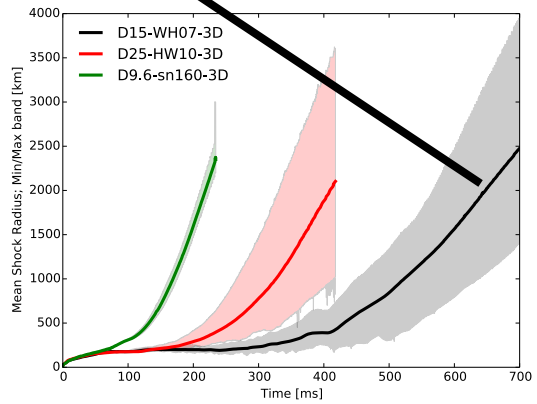






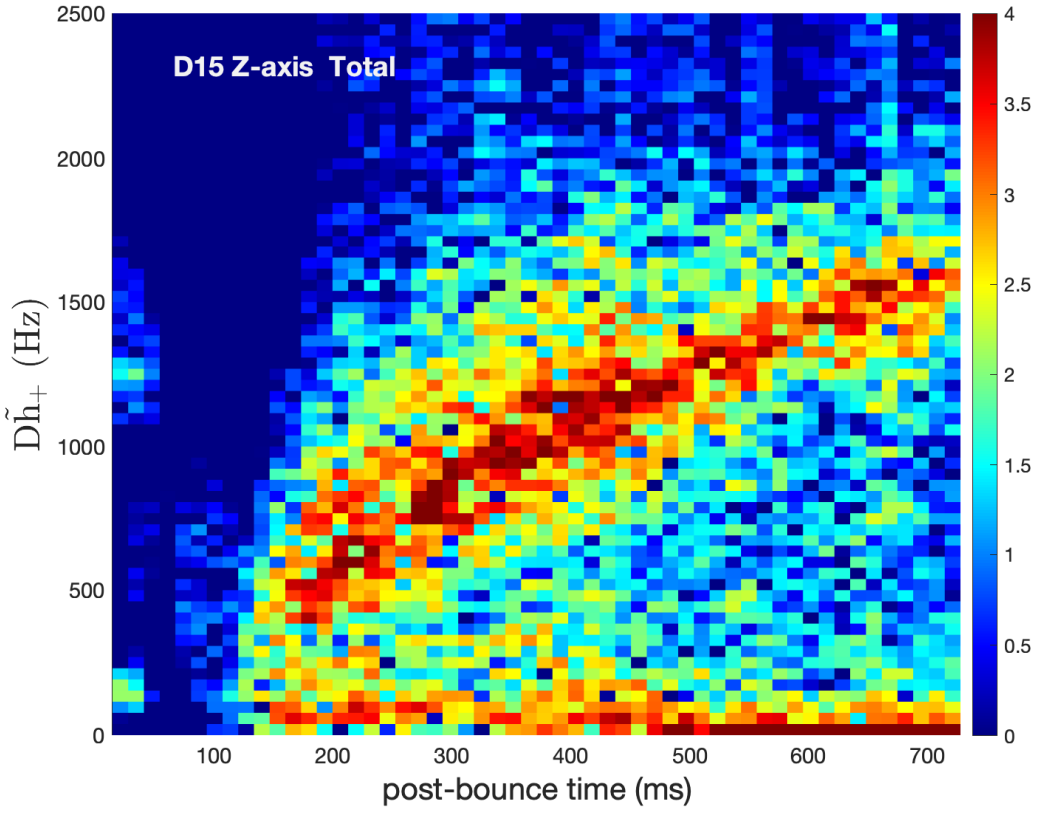


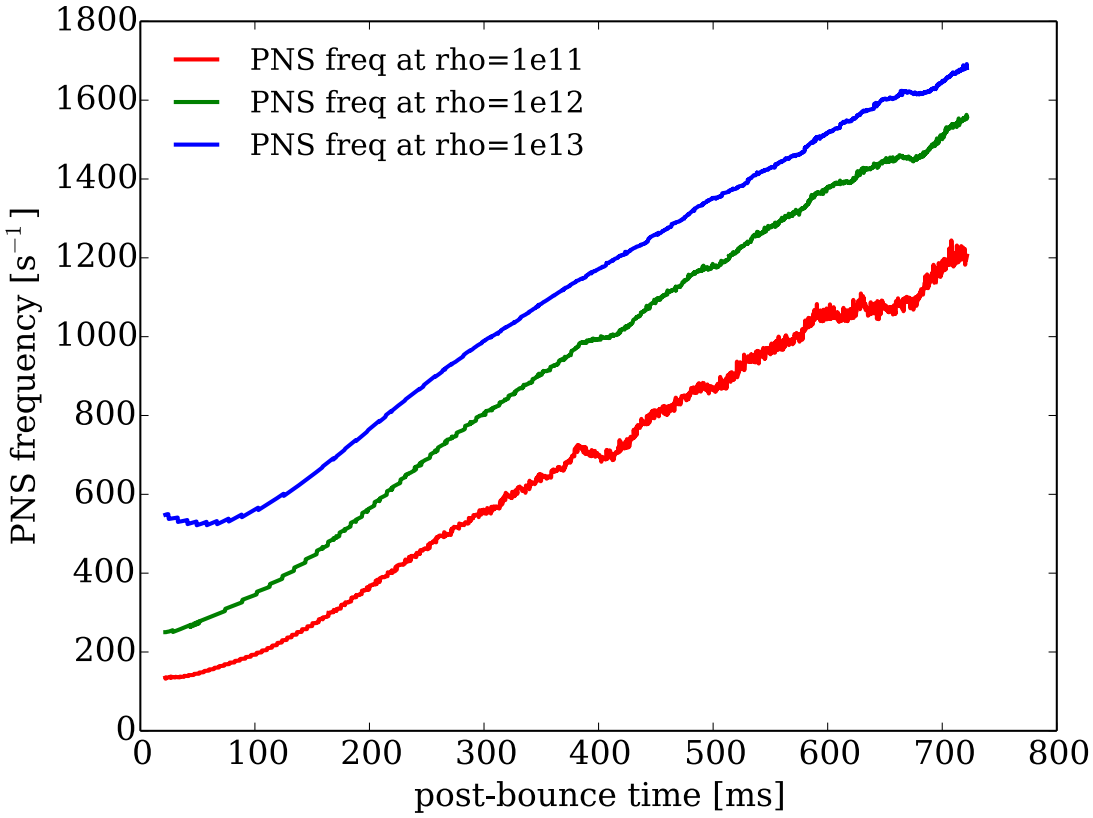




Mezzacappa, Marronetti, Landfield, Lentz, et al. 2022, arXiv 2208.10643

Culling Information about the Proto-Neutron Star





$$f_p = \frac{1}{2\pi}\frac{GM}{R^2}\frac{1}{c_s}\sqrt{\Gamma - 1}\left(1 - \frac{GM}{Rc^2}\right)^{3/2}$$

Mueller, Janka, and Marek, *Ap.J.* **766** 43 (2013)

Outlook

Gravitational waves and neutrinos are the only “messengers” that will bring us **direct** information about the core collapse supernova central engine:

- Progenitor Mass and Rotation
- Existence of Neutrino-Driven Convection
- Existence of the SASI
- Existence of Proto-Neutron Star (PNS) Instabilities and the Properties of the PNS and its Evolution
- High-Density, Neutron-Rich Nuclear Equation of State in the PNS

A Galactic event within $O(10)$ kpc would be detectable, as would the neutrino signal. Szczepanczyk et al. PRD 104 102002 (2021)

*The **Volunteer** supernova group is well positioned to provide theoretical input in preparation for this watershed event.*

Third-Generation detectors will increase the distance at which a core collapse supernova gravitational wave signal is detectable, to $O(100)$ kpc.

Our fundamental limitation right now is the low Galactic core collapse supernova event rate. A rate of 1 core collapse supernova per year would require an additional two orders of magnitude increase in sensitivity beyond what is planned for third-generation detectors. Srivistava et al. PRD **100**, 043026 (2019)